

Extremal problems on integer polynomials via complex function theory

by

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We discuss several old problems on polynomials with integer coefficients that contain interesting components related to complex function theory. The first one is the integer Chebyshev problem about integer polynomials with small norms, originated in the work of Hilbert, Pólya, Schur, Fekete, and others. While this problem remains open even for intervals of the real line, we present its solution for some classes of lemniscates defined by irreducible polynomials. A related group of questions concerns roots of integer polynomials, and includes the conjectures of Lehmer and Schinzel-Zassenhaus about location of such roots with respect to the unit circle. In addition to the classical forms of those conjectures, we also consider their analogues for lemniscates. All of these problems are treated by a mix of methods from complex analysis, potential theory and number theory.