On Ahlfors' Schwarzian derivatives in Euclidean Space

by

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We discuss Ahlfors' Schwarzian derivatives for curves in euclidean space introduced in [Ah] some thirty years ago. The definitions consider separate generalizations of the real and imaginary part of the classical operator in the complex plane, and have important invariance properties with respect to the Möbius group in $\mathbb{R}^n$. We describe applications of the real Schwarzian to the study of simple curves in $\mathbb{R}^n$ and infinite ends, to knots in $\mathbb{R}^3$, as well as to the injectivity of the conformal parametrization of minimal surfaces in 3-space. The role of the imaginary Schwarzian will be presented in $\mathbb{R}^3$, highlighting its connection with the osculating sphere, a new transformation law under Möbius transformations, and theorems on the existence and uniqueness of parametrized curves with prescribed real and imaginary Schwarzians.

[Ah] L. V. Ahlfors, *Cross-ratios and Schwarzian derivatives in $\mathbb{R}^n$*, Complex Analysis: Articles dedicated to Albert Pfluger on the occasion of his 80th birthday, Birkhäuser Verlag, Basel, 1989, 1-15.

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