UNIVERSIDAD TECNICA FEDERICO SANTA MARIA

Computational Methods and Function Theory 2021

Computational Methods And Function Theory

January 10-14, 2022
Virtual Conference
Valparaíso, Chile

In memoriam

STEPHAN RUSCHEWEYH

1944-2019

UNIVERSIDAD TECNICA FEDERICO SANTA MARIA

Cmft
Computational Methods and Function Theory

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VIRTUAL CONFERENCE

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Stephan Ruscheweyh, 1944-2019

Universidad Técnica Federico Santa María
Valparaíso, Chile

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Bohr radius for Banach spaces on simply connected domains

Vasudevarao Allu and Himadri Halder

Abstract

Let $H^\infty(\Omega, X)$ be the space of bounded analytic functions $f(z) = \sum_{n=0}^{\infty} x_n z^n$ from a proper simply connected domain Ω containing the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ into a complex Banach space X with $\|f\|_{H^\infty(\Omega,X)} \leq 1$. Let $\phi = \{\phi_n(r)\}_{n=0}^{\infty}$ with $\phi_0(r) \leq 1$ such that $\sum_{n=0}^{\infty} \phi_n(r)$ converges locally uniformly with respect to $r \in [0, 1)$. For $1 \leq p, q < \infty$, we denote

$$R_{p,q,\phi}(f, \Omega, X) = \sup\left\{ r \geq 0 : \|x_0\|^p \phi_0(r) + \left(\sum_{n=1}^{\infty} \|x_n\| \phi_n(r) \right)^q \leq \phi_0(r) \right\}$$

and define the Bohr radius associated with ϕ by

$$R_{p,q,\phi}(\Omega, X) = \inf\left\{ R_{p,q,\phi}(f, \Omega, X) : \|f\|_{H^\infty(\Omega,X)} \leq 1 \right\}.$$

In this article, we extensively study the Bohr radius $R_{p,q,\phi}(\Omega, X)$, when X is an arbitrary Banach space and $X = \mathcal{B}(\mathcal{H})$ is the algebra of all bounded linear operators on a complex Hilbert space $\mathcal{H}$. Furthermore, we establish the Bohr inequality for the operator-valued Cesáro operator and Bernardi operator.

Subject Classification: Primary 46E40, 47A56, 47A63; Secondary 46B20, 30B10, 30C20, 30C65
keywords: Banach space, operator valued; Simply connected domains; Bohr radius; Cesáro operator, Bernardi operator

Widom factors for generalized Jacobi measures

by

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Let K be a regular compact subset of $[-1,1]$ and μ_K be its equilibrium measure. For a given weight w on K, let $T_{n,w}^{(K)}$ denote the n-th weighted Chebyshev polynomial with respect to w on K and $P_n(\cdot;\mu)$ denote the n-th monic orthogonal polynomial for a finite Borel measure μ with $\mathrm{supp}(\mu) = K$. Define

$$W_{\infty,n}(K,w) := \frac{\|w\, T_{n,w}^{(K)}\|_K}{\mathrm{Cap}(K)^n} \tag{1}$$

and

$$W_{2,n}(\mu) := \frac{\|P_n(\cdot;\mu)\|_{L_2(\mu)}}{\mathrm{Cap}(K)^n}. \tag{2}$$

We discuss optimal upper and lower bounds for $W_{\infty,n}(K,w)$ if $w(x) = \sqrt{1-x^2}, \sqrt{1-x}$ or $\sqrt{1+x}$. We also investigate optimal lower bounds for $W_{2,n}(\mu)$ for the cases $d\mu(x) = (1-x^2)d\mu_K(x)$, $d\mu(x) = (1-x)d\mu_K(x)$ and $d\mu(x) = (1+x)d\mu_K(x)$.

On Hilbert lemniscate theorem for a system of continua.

by

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Let K be a compact set in the complex plane consisting of a finite number of continua. We study the rate of approximation of K from the outside by lemniscates in terms of level lines of the Green function for the complement of K.

ON SPECTRUM OF A SELFADJOINT DIFFERENCE OPERATOR ON THE GRAPH-TREE

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We consider a class of the selfadjoint discrete Schrödinger operators defined on an infinite homogeneous rooted graph-tree. The potential of this operator consists of the coefficients of the Nearest Neighbor Recurrence Relations (NNRRs) for the Multiple Orthogonal Polynomials (MOPs).

For the general class of potentials, generated by Angelesco MOPs we prove that the essential spectrum of these operators is a union of the supports of the components of the vector orthogonality measure $\vec{\mu} := (\mu_1, \dots, \mu_d)$ for the Angelesco MOPs. It is a joint work with Sergey Denisov (Madison University) and Maxim Yattselev (IUPUI).

Surface identification through rational approximation of the back-scattering of an electromagnetic plane wave

by

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By measuring the scattered electromagnetic field produced by a plane wave on a smooth object at various frequencies, and then performing rational or meromorphic approximation of the transfer function, we consider the issue of identifying the shape of the object from the recovery of some characteristic singularities (which are poles because the object is smooth, but typically infinite in number). This technique can also be used to identify certain physical characteristics of the object for nondestructive testing.

Joint work with L. Baratchart, J. Leblond, M. Olivi and F. Seyfert

Analytic approximation and Carleman formulas

by

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On the unit disk $\mathbb{D}$, for I a subset of the unit circle $\mathbb{T}$, Carleman formulas are of the form

$$f_\alpha(z) = \frac{1}{2i\pi w^\alpha(z)} \int_I w^\alpha(\xi)\, f(\xi) \frac{d\xi}{\xi - z}, \qquad z \in \mathbb{D}, \tag{1}$$

with w an outer function such that $|w| > 1$ on I and 1 on $J := \mathbb{T} \setminus I$. It is known after work by Goluzin-Krylov and Patil that if f is the trace of an analytic function in the disk, say, of Hardy class H^p, $1p < \infty$, then f_α converges to f. In this talk we discuss more generally formulas of the form

$$g_\rho(f, \Psi)(z) = \frac{1}{2i\pi} \int_{\mathbb{T}} \frac{w(\xi)}{w(z)} \,(f \vee \Psi)(\xi)\, \frac{d\xi}{\xi - z}, \tag{2}$$

where f needs not be the trace of an analytic function, $f \vee \Psi$ indicates concatenation of f defined on I and Ψ defined on J, while w is an outer function that needs not necessarily have modulus 1 on J. These functions have surprising extremal properties, and we discuss their computation and continuity properties.

Joint work with J. Mashreghi

On some holomorphic function spaces on the ball

by

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Let B^n, $n \in \mathbb{N}$, be the unit ball in $\mathbb{C}^n$, with the Bergman metric. For $k \in \mathbb{N}$ and a discrete subgroup Γ of $Aut(B^n)$ there is a corresponding Hilbert space of holomorphic functions on B^n, sometimes called the space of automorphic forms. With a subset of the ball, X, satisfying some not too restrictive conditions, one can associate a sequence of these holomorphic functions (f_k). There is an interesting interplay between the geometry of X and the properties of the sequence (f_k) (e.g. the $k \to \infty$ asymptotics of the norms). I will present general results as well as computations for specific choices of X. This is mostly based on [AB, B].

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L_p Markov exponent of certain UPC sets

by

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We say that a compact set $\emptyset \neq E \subset \mathbb{R}^m$ satisfies L_p Markov type inequality (or: is a L_p Markov set) if there exist $\kappa, C > 0$ such that, for each polynomial $P \in \mathcal{P}(\mathbb{R}^m)$ and each $\alpha \in \mathbb{N}_0^m$,

$$\|D^\alpha P\|_{L_p(E)} \leq (C(\deg P)^\kappa)^{|\alpha|} \|P\|_{L_p(E)}, \tag{1}$$

where $D^\alpha P = \frac{\partial^{|\alpha|} P}{\partial x_1^{\alpha_1} \ldots \partial x_m^{\alpha_m}}$ and $|\alpha| = \alpha_1 + \cdots + \alpha_m$.

Clearly, by iteration, it is enough to consider in the above definition multi-indices α with $|\alpha| = 1$. The inequality (1) is a generalization of the classical Markov inequality:

$$\|P'\|_{C([-1,1])} \leq (\deg P)^2 \|P\|_{C([-1,1])}.$$

The classical Markov inequality and its extensions for multivariate case play a central role in various approximation problems (see [7], [8]).

In this talk we shall consider the following problem:

For a given L_p Markov set E determine $\mu_p(E) := \inf\{\kappa : E$ satisfies (1)$\}$.

The quantity $\mu_p(E)$ is called L_p Markov exponent and was first considered by Baran and Pleśniak in [2] for $p = \infty$. For any compact set E in $\mathbb{R}^m$ we have $\mu_p(E) \geq 2$. It is well known that for Locally Lipschitzian compact subsets of $\mathbb{R}^m$ and thus, in particular convex domains the L_p Markov exponent is equal to 2 (see [3]). If $E \subset \mathbb{R}^m$ is a *Lip*γ, $0 < \gamma < 1$ cuspidal domain, then $\mu_\infty(E) = \frac{2}{\gamma}$ (see for instance, [4], [1], [5]). Recently, in [6], for $1 \leq p < \infty$, the same exponent $\frac{2}{\gamma}$ as in L_∞ norm was obtained for *Lip*γ, $0 < \gamma < 1$ cuspidal piecewise graph domains that are imbedded in an affine image of the l_γ ball having one of its vertices on the boundary. Our goal is to establish L_p Markov exponent of the following domains

$$K := \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1,\, ax^k \leq y \leq f(x)\},$$

where $k \in \mathbb{N}$, $k \geq 2$, $a > 0$ and $f : [0,1] \to [0,\infty)$ is a convex function such that $f(1) > a$, $f'(0) = f(0) = 0$, $f'(1) < \infty$, and $(f)^{1/k}$ is a concave function on the interval $(0,1)$.

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Fast conformal mapping via computational and hyperbolic geometry

by

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The conformal map from the unit disk to the interior of a polygon P is given by the Schwarz-Christoffel formula, but this is stated in terms of parameters that are hard to compute from P. After some background and motivation, I explain how the medial axis of a domain, a concept from computational geometry, can be used to give a fast approximation to these parameters, with bounds on the accuracy that are independent of P. The precise statement involves quasiconformal mappings, and proving these bounds uses a result about hyperbolic convex sets originating in Thurston's work on 3-manifolds. If time permits, I will mention some applications to optimal meshing and triangulation of planar polygons.

A new necessary condition for maximal convergence on unconnected sets

by

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Let f be holomorphic on the compact set $E \subset \mathbb{C}$ with regular complement $\Omega = \overline{\mathbb{C}} \setminus E$. A theorem of Grothmann states that interpolating polynomials to f are maximally convergent if a subsequence of the interpolation points converges to the equilibrium distribution of E in the weak* sense, provided f is not an entire function. For obtaining a complete proof of this theorem, even for unconnected sets E, we apply a new necessary condition for maximal convergence:

Let

$$E_\sigma := \left\{ z \in \Omega \,:\, G(z) < \log \sigma \right\} \cup E$$

be the *Green domain* with boundary $\Gamma_\sigma = \partial E_\sigma$. Since Ω is regular, the Green domain E_σ consists of a finite number of Jordan regions E_σ^i which are mutually exterior,

$$E_\sigma = E_\sigma^1 \cup E_\sigma^2 \cup \cdots \cup E_\sigma^{l_\sigma}\,, \qquad l_\sigma \in \mathbb{N}\,.$$

Let f be holomorphic on E and let $\rho(f)$ denote the maximal parameter ρ such that f is holomorphic on E_ρ. Then the polynomials $\left\{p_n\right\}_{n\in\mathbb{N}}$ with $p_n n \in \mathcal{P}_n$ converge maximally to f if and only if

$$\frac{\sigma}{\rho(f)} = \limsup_{n\to\infty} \left\|f - p_n\right\|_{\Gamma_\sigma}^{1/n} = \limsup_{n\to\infty} \min_{1\le i\le l_\sigma} \left\|f - p_n\right\|_{\Gamma_\sigma^i}^{1(n}$$

for any σ, $1 < \sigma < \rho(f) < \infty$, where $\Gamma_\sigma^i = \partial E_\sigma^i$.

Accessory parameters for four-punctured spheres

by

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I will show that the accessory parameter associated to the uniformization of a four-punctured sphere is the unique zero of a system of infinitely many equations. The proof exploits the modularity of the solution of the uniformizing differential equation and the existence of a non-trivial group of automorphisms for every four-punctured sphere. This result gives a method for the numerical computation of the uniformizing parameter. As an application, I will present a numerical study of the real-analytic map associating to a four-punctured sphere its uniformizing parameter, and make some remarks on the size of the coefficients of its local expansion.

Canonical embeddings of pairs of arcs

by

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I'll discuss recent joint work with A. Eremenko. We showed that for given four points in the Riemann sphere and a given isotopy class of two disjoint arcs connecting these points in two pairs, there exists a unique configuration with the property that each arc is a hyperbolic geodesic segment in the complement of the other arc. The configuration can explicitly be described with elliptic functions.

Computation and analysis of the asymptotic behaviour of the compound means

by

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We derive and examine algorithms for computing asymptotic expansion of the composite mean of two arbitrary means M and N. Then we analyse asymptotic behaviour of the compound mean $M \otimes N$. Examples and application to some classical means are also presented.

Quasi-Hadamard Product of Sakaguchi-Type Functions Defined By $q-$Difference Operator

by

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In this paper, we introduce two Sakaguchi-type classes of analytic functions which are starlike and convex functions with respect to symmetric points involving $q-$difference operator in the open unit disc. We find sufficient coefficient estimates for the functions belonging to these classes. By applying these coefficient estimates, we get quasi-Hadamard product for such functions.

On Ahlfors' Schwarzian derivatives in Euclidean Space

by

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We discuss Ahlfors' Schwarzian derivatives for curves in euclidean space introduced in [Ah] some thirty years ago. The definitions consider separate generalizations of the real and imaginary part of the classical operator in the complex plane, and have important invariance properties with respect to the Möbius group in $\mathbb{R}^n$. We describe applications of the real Schwarzian to the study of simple curves in $\mathbb{R}^n$ and infinite ends, to knots in $\mathbb{R}^3$, as well as to the injectivity of the conformal parametrization of minimal surfaces in 3-space. The role of the imaginary Schwarzian will be presented in $\mathbb{R}^3$, highlighting its connection with the osculating sphere, a new transformation law under Möbius transformations, and theorems on the existence and uniqueness of parametrized curves with prescribed real and imaginary Schwarzians.

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Joint work with Peter Duren, Julian Gevirtz and Brad Osgood

On weighted classes of analytic and subharmonic functions in the unit disc

by

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In [1] J. Bruna and J. Ortega-Cerdá described measures which are Riesz measures for subharmonic functions $u \in L^p(\Omega)$ for a large class of domains in $\mathbb{R}^n$. As a consequence, zero sets of analytic functions f with $\log|f| \in L^p(\mathbb{D})$ are characterized. We generalize this result for $L^p_\omega(\mathbb{D})$ for a class of weights ω containing the functions $\omega(r) = (1-r)^\gamma$, $\gamma \in (-1, +\infty)$.

Besides, we give conditions under which $u \in L^p_\omega(\mathbb{D})$ is equivalent to $u^+ \in L^p_\omega(\mathbb{D})$, where $u^+ = \max\{u, 0\}$, u is subharmonic in $\mathbb{D}$.

Joint work with Jouni Rättya.

[1] J. Bruna, J. Otrega-Cerdá, On L^p-solutions of the Laplace equation and zeros of holomorphic functions, Annali della Scuola Normale Superiore di Pisa – Classe di Scienze **24** (1997), no. 3, 571–591.

The Beurling-Lax-Halmos Theorem for Infinite Multiplicity

Raúl E. Curto*, In Sung Hwang and Woo Young Lee

Abstract. In this paper we consider several questions emerging from the Beurling-Lax-Halmos Theorem, which characterizes the shift-invariant subspaces of vector-valued Hardy spaces. The Beurling-Lax-Halmos Theorem states that a backward shift-invariant subspace is a model space $\mathcal{H}(\Delta) \equiv H^2_E \ominus \Delta H^2_E$, for some inner function Δ. Our first question calls for a description of the set F in H^2_E such that $\mathcal{H}(\Delta) = E^*_F$, where E^*_F denotes the smallest backward shift-invariant subspace containing the set F. In our pursuit of a general solution to this question, we are naturally led to take into account a canonical decomposition of operator-valued strong L^2-functions. Next, we ask: Is every shift-invariant subspace the kernel of a (possibly unbounded) Hankel operator ? Consideration of the question on the structure of shift-invariant subspaces leads us to study and coin a new notion of "Beurling degree" for an inner function. We then establish a deep connection between the spectral multiplicity of the model operator (the truncated backward shift) and the Beurling degree of the corresponding characteristic function. At the same time, we consider the notion of meromorphic pseudo-continuations of bounded type for operator-valued functions, and then use this notion to study the spectral multiplicity of model operators between separable complex Hilbert spaces. In particular, we consider the case of multiplicity-free: more precisely, for which characteristic function Δ of the model operator T does it follow that T is multiplicity-free, i.e., T has multiplicity 1 ? We show that if Δ has a meromorphic pseudo-continuation of bounded type in the complement of the closed unit disk and the adjoint of the flip of Δ is an outer function, then T is multiplicity-free.

Talk based on an article, with the same title, accepted for publication in *J. Funct. Anal.*

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* Presenter

2010 *Mathematics Subject Classification.* Primary 46E40, 47B35, 30H10, 30J05; Secondary 43A15, 47A15

Key words. The Beurling-Lax-Halmos Theorem, strong L^2-functions, a canonical decomposition, a complementary factor of an inner function, the degree of non-cyclicity, the Beurling degree, the spectral multiplicity, the model operator.

The Bohr phenomenon for operator-valued holomorphic functions

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In 1914, Harald Bohr proved that for any holomorphic self mapping $f(z) = \sum_{n=0}^{\infty} a_n z^n$ of the open unit disk $\mathbb{D}$,

$$\sum_{n=0}^{\infty} |a_n| r^n \leq 1 \tag{1}$$

for all $z \in \mathbb{D}$ with $|z| = r \leq 1/6$. Inequalities of the type (1) are commonly known as *Bohr inequalities* nowadays, and appearance of such inequalities in a result is generally termed as the *Bohr phenomenon*. The above theorem was re-established with the best possible constant 1/3 instead of 1/6 by Wiener, Riesz and Schur independently. In the last few decades, the study of the Bohr phenomenon has grown to be a popular area of research. In particular, the Bohr inequality has been extended for holomorphic functions that map $\mathbb{D}$ inside some domain $\Omega \subsetneq \mathbb{C}$ other than $\mathbb{D}$. In the first half of this talk, we will present a couple of results of the above kind and show that a proper combination of some methods from geometric function theory and operator theory yields analogous results for operator-valued holomorphic functions defined in $\mathbb{D}$.

In the second part of our discussion, we concentrate on a generalization of Bohr's result by Enrico Bombieri, which stated that for any holomorphic self mapping $f(z) = \sum_{n=0}^{\infty} a_n z^n$ of $\mathbb{D}$, the inequality (1) is true for

$$|z| = r \leq \begin{cases} 1/(1 + 2|a_0|) \text{ if } |a_0| \geq 1/2, \\ \sqrt{(1/2)(1 - |a_0|)} \text{ if } |a_0| \leq 1/2. \end{cases}$$

When $|a_0| \geq 1/2$, the quantity $1/(1 + 2|a_0|)$ is the best possible. We will show that this result admits an extension in operator-valued framework, and discuss some of its consequences.

This talk is mostly based on the following articles:

- B. Bhowmik, N. Das: Bohr phenomenon for operator-valued functions, *Proceedings of the Edinburgh Mathematical Society*, 64 (2021), no. 1, 72–86.
- B. Bhowmik, N. Das: An operator-valued analogue of a result of Bombieri (communicated, an earlier version is available at: arXiv:2003.05810v1 [math.CV]).

On one inequality of M. A. Lavrentyev

by

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The result of M. A. Lavrentyev on the product of conformal radii of two non-overlapping simply connected domains is generalized and strengthened. A method that allowed to obtain new estimates of the products of the inner radii of mutually non-overlapping domains with respect to the fixed points of the complex plane is proposed.

This work was supported by the budget program "Support of the development of priority trends of scientific researches" (KPKVK 6541230).

Bounded Point Derivations and Functions of Bounded Mean Oscillation

by

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Given a subset X of the complex plane and a family of functions on this set, it is often the case that these functions may have a greater degree of smoothness at the boundary than would otherwise be expected. One measure of this is the existence of a bounded point derivation, which is a bounded linear functional that extends the concept of the derivative to functions which may not be differentiable. Necessary and sufficient conditions depending only on the structure of X have been determined for important families of functions, most notably $R(X)$, the uniform closure of rational functions with poles off X. Recently we determined necessary and sufficient conditions for the existence of bounded point derivations on $A_0(X)$, the family of VMO functions that are analytic in a neighborhood of X. In this talk we will describe these conditions and give examples of sets with these kinds of bounded point derivations.

Some results of Fekete - Szegö type for Bavrin's families of holomorphic functions in $\mathbb{C}^n$

by

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In the lecture will be considered for $\lambda \in \mathbb{C}, k \geq 2$, the problem of the sharp upper estimate

$$\mu_{\mathcal{G}}(Q_{f,2k} - \lambda\,(Q_{f,k})^2) \leq M(\lambda, k)$$

for the pairs $Q_{f,k}$ and $Q_{f,2k}$ homogeneous polynomials of functions belonging to some Bavrin's [1] families $\mathcal{M}_{\mathcal{G}}$, $\mathcal{N}_{\mathcal{G}} \subset \mathcal{H}_{\mathcal{G}}(1)$ which elements satisfy also a (j, k)-symmetry condition.

Here $\mathcal{G} \subset \mathbb{C}^n, n \geq 1$, is bounded complete n-circular domain, $\mu_{\mathcal{G}}(Q_m)$ means a Minowski balance of m-homogeneous polynomial, $\mathcal{H}_{\mathcal{G}}(1)$ is a collection of holomorphic functions $f : \mathcal{G} \to \mathbb{C}$, normalized by the condition $f(0) = 1$.

These families $\mathcal{M}_{\mathcal{G}}$, $\mathcal{N}_{\mathcal{G}}$ are defined by the following family $\mathcal{C}_{\mathcal{G}}$,

$$\mathcal{C}_{\mathcal{G}} = \{f \in \mathcal{H}_{\mathcal{G}}(1) : \operatorname{Re} f(z) > 0, z \in \mathcal{G}\}$$

and by the following Temljakov [4] linear operator $\mathcal{L} : \mathcal{H}_{\mathcal{G}} \longrightarrow \mathcal{H}_{\mathcal{G}}$

$$\mathcal{L}f(z) = f(z) + Df(z)(z), z \in \mathcal{G},$$

where $Df(z)$ means the Frěchet derivative of f at the point z.

The definitions of Bavrin's families $\mathcal{M}_{\mathcal{G}}$, $\mathcal{N}_{\mathcal{G}}$ correspond to geometric properties of univalent functions of a complex variable, like as starlikeness and convexity.

The mentioned estimate is a generealization of the well known planar Fekete-Szegö [2] result onto the several complex variable case.

As application of these results there is given the solution of a Fekete-Szegö type problem for a family of normalized biholomorphic starlike mappings in $\mathbb{C}^n$. All presented results are from the paper [3].

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Joint work with Piotr Liczberski Institute of Mathematics
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Approximation of analytic functions of several variables by multidimensional A- and J-fractions with independent variables

by

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Let N be a fixed natural number and

$$\mathcal{I} = \{i(k) : \ i(k) = (i_1, i_2, \ldots, i_k), \ 1 \le i_p \le i_{p-1}, \ 1 \le p \le k, \ i_0 = N\}$$

be the sets of multiindices.

We consider the problem of approximation of analytic functions of several variables by multidimensional A-fractions with independent variables

$$\sum_{i_1=1}^{N} \frac{p_{i(1)} z_{i_1}}{1 + q_{i(1)} z_{i_1} +} \sum_{i_2=1}^{i_1} \frac{(-1)^{\delta_{i_1,i_2}} p_{i(2)} z_{i_1} z_{i_2}}{1 + q_{i(2)} z_{i_2}} + \sum_{i_3=1}^{i_2} \frac{(-1)^{\delta_{i_2,i_3}} p_{i(3)} z_{i_2} z_{i_3}}{1 + q_{i(3)} z_{i_3}} + \cdots$$

and multidimensional J-fractions with independent variables

$$\sum_{i_1=1}^{N} \frac{p_{i(1)}}{q_{i(1)} + z_{i_1} +} \sum_{i_2=1}^{i_1} \frac{(-1)^{\delta_{i_1,i_2}} p_{i(2)}}{q_{i(2)} + z_{i_2}} + \sum_{i_3=1}^{i_2} \frac{(-1)^{\delta_{i_2,i_3}} p_{i(3)}}{q_{i(3)} + z_{i_3}} + \cdots,$$

where $p_{i(k)} \in \mathbb{C} \setminus \{0\}$, $q_{i(k)} \in \mathbb{C}$ for $i(k) \in \mathcal{I}$, $\delta_{i,j}$ is a Kronecker symbol, $(z_1, z_2, \ldots, z_N) \in \mathbb{C}^N$.

An algorithm for the expansion of the formal multiple power series into the corresponding multidimensional A-fraction with independent variables is constructed and investigated. The connection between this branched continued fraction and multidimensional J-fraction with independent variables is shown. Some questions of convergence of the above-mentioned branched continued fractions are considered.

Several numerical experiments show the efficiency, power, and feasibility of using the branched continued fractions to numerically approximate analytic functions of several variables from their formal multiple power series.

On the Best Uniform Polynomial Approximation to the Checkmark Function

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Abstract

The best uniform polynomial approximation of the checkmark function $f(x) = |x - \alpha|$ is considered, as α varies in $(-1, 1)$. For each fixed degree n, the minimax error $E_n(\alpha)$ is shown to be piecewise analytic in α. In addition, $E_n(\alpha)$ is shown to feature $n-1$ piecewise linear decreasing/increasing sections, called V-shapes. The points of the alternation set are proven to be piecewise analytic and monotone increasing in α and their dynamics are completely characterized. We also prove a conjecture of Shekhtman that for odd n, $E_n(\alpha)$ has a local maximum at $\alpha = 0$.

* *Joint work with Alan Legg - PFW and Ramon Orive - University La Laguna*

CRITERIA FOR UNIVALENCE AND QUASICONFORMAL EXTENSION FOR HARMONIC MAPPINGS ON PLANAR DOMAINS

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The subject of this talk is the extension of the theory of the Schwarzian derivative

$$Sf = \left(\frac{f''}{f'}\right)' - \frac{1}{2}\left(\frac{f''}{f'}\right)^2$$

to planar harmonic mappings. We give criteria for univalence, homeomorphic and quasiconformal extension successively on quasidisks, finitely connected domains (all of whose boundary components are either points or quasicircles) and on uniform domains; see [1] and [2]. Moreover, we give two explicit quasiconformal extensions for harmonic mappings in the unit disk, both of which are generalizations of the Ahlfors-Weill extension for holomorphic functions; see [3]. All these results are based on the definition of the Schwarzian derivative for harmonic mappings introduced by Hernández and Martín [4].

Note: Please find in continuation on this PDF file the papers [1], [2] and [3].

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Uniqueness Theorems for Fourier Quasicrystals and Temperate Distributions with Discrete Support

by

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We prove that two Fourier quasicrystals on the Euclidean space $\mu = \sum_n a(\lambda_n)\delta_{\lambda_n}$, $\nu = \sum_n b(\gamma_n)\delta_{\gamma_n}$ with supports of bounded density, for which $\lambda_n - \gamma_n \to 0$ and $a(\lambda_n) - b(\gamma_n) \to 0$ as $n \to \infty$, coincide identically. In fact, we can check the convergence only for certain rare subsequences of pairs λ_n, γ_n.

We also extend the uniqueness theorem to temperate distributions with discrete support.

An extremal problem for polynomials over the unit disc of the complex plane

by

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We consider the sum S of the absolute values of the first two coefficients of a polynomial p over the class P_d of all complex polynomials of degree at most d and observe that in spite of clever work by Brickman, Rahman, Ruscheweyh, Shapiro and others, there is no known numerical upper bound for S/M over P_d that is sharp for infinitely many indices d where M is the maximum modulus of the polynomial p over the unit disc of the complex plane. However we shall exhibit a sharp upper bound for $S/M(d)$ where $M(d)$ is the maximum modulus of the polynomial p over a disc centered at the origin but of radius larger than unity depending on d.

Harmonic extension through cylindrical and conical surfaces

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The Schwarz reflection principle provides a simple formula for extending a harmonic function h on a domain $\omega \subset \mathbb{R}^N$ through a relatively open subset E of $\partial\omega$ on which h vanishes, provided E lies in a hyperplane. A related reflection formula holds when E lies in a sphere. Indeed, when $N = 2$, a reflection formula holds whenever E is contained in an analytic arc. However, when $N \geq 3$ and N is odd, Ebenfelt and Khavinson [1] (cf. Chapter 12 of [8]) have shown that a point-to-point reflection law holds only when the containing real analytic surface is either a hyperplane or a sphere. Thus other ideas are needed to investigate the extension properties of harmonic functions that vanish on other types of set E.

In this direction the following problem was posed by Dima Khavinson at various international conferences: if h is harmonic on an infinite cylinder and vanishes on the boundary, does it extend harmonically to all of $\mathbb{R}^N$? In the planar case, where h is harmonic on an infinite strip, a positive answer follows readily by repeated application of the Schwarz reflection principle. In higher dimensions the problem was eventually also shown to have an affirmative answer [2] by analysis of the Green function of the cylinder. Thus we have a solution to one non-trivial case of the following general problem, where the set E is assumed to lie in a real analytic surface.

Problem 1 For a domain ω in $\mathbb{R}^N$ and a subset E of $\partial\omega$ identify a larger domain ω_E such that each harmonic function on ω which vanishes continuously on E has a harmonic extension to ω_E.

This problem can be challenging even when E is contained in the zero set of some polynomial. For example, the cylindrical case corresponds to the polynomial $(x', x_N) \mapsto \|x'\|^2 - 1$, where $x' \in \mathbb{R}^{N-1}$. This talk will describe several results in this direction where E is contained in a cylindrical or conical surface. The various domain reflection results that arise are striking, given that reflection formulae for the harmonic functions themselves are known not to exist.

Currently, our methods rely on detailed calculations involving Bessel and Legendre functions. Our hope is that the outcome of these specific studies will suggest a more general approach to Problem 1.

This is joint work with Hermann Render ([2] - [7]).

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Existence of the corona

by

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The existence and uniqueness of the Shilov boundary of a commutative unital Banach algebra was shown to exist by Shilov himself. For a bounded domain $\Omega \subset \mathbb{C}^n$, the Shilov boundary of Ω is defined to be the Shilov boundary of the function algebra $A(\Omega)$. Physicists are also interested in the Shilov boundary for unbounded domains, but for some mathematicians (guess whom), the definition, existence and uniqueness are not obvious. These questions are related to the existence of a corona for a related function algebra.

Critically fixed anti-rational maps and gravitational lensing

by

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Gravitational lensing by point masses, situated in a common plane perpendicular to the line between light source and observer, is intimately connected to fixed point equations and dynamics of anti-rational maps (i.e., complex conjugates of rational maps.) In 2003, Rhie gave examples to show that a gravitational lens consisting of $n \geq 2$ point masses can produce $5n-5$ apparent images of a single light source, and in 2006 Khavinson and Neumann used techniques from complex dynamics to prove Rhie's conjecture that $5n-5$ is actually the maximal number of apparent images produced by such gravitational lenses. In light of these results, an n-point gravitational lens producing $5n-5$ apparent images is called an *maximal lensing configuration.*

I will explain a recently established complete classification of critically fixed anti-rational maps, and how it leads to a partial classification of equivalence classes of maximal lensing configurations by certain planar graphs. In particular, I will give explicit examples of maximal lensing configurations which were not previously known and are not equivalent to Rhie's examples.

A curious difference between $H(\mathbb{D})$ and $H(\mathbb{C})$

by

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It is a classical result of G. R. MacLane that the differentiation operator $Df = f'$ has a dense orbit when viewed as an operator on the space $H(\mathbb{C})$ of entire functions; in other words, it is hypercyclic. The same is then true if one regards D as an operator on the space $H(\mathbb{D})$ of holomorphic functions on the unit disk. More recent research has shown that the operator is even chaotic and frequently hypercyclic. If one widens the perspective then a curious difference appears between the spaces $H(\mathbb{D})$ and $H(\mathbb{C})$. Indeed, the differentiation operator is a special weighted shift operator. Now, it turns out that, on one of the two spaces, every frequently hypercyclic weighted shift is chaotic, while on the other one there are weighted shifts that are frequently hypercyclic but not chaotic. Which is the pathological space, and why?

Joint work with Stéphane Charpentier and Quentin Menet

Ax–Schanuel type theorems on functional transcendence via Nevanlinna theory

by

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We say a function F has an Ax-Schanuel property if for any $\mathbb{Q}$-linearly independent modulo $\mathbb{C}$ entire functions of one complex variable $f_1, \ldots, f_n$, the transcendence degree over $\mathbb{C}$ of $f_1, \ldots, f_n, F(f_1), \ldots, F(f_n)$ is at least $n+1$. The Ax-Schanuel Theorem implies that the exponential function $e(z) := e^{2\pi i z}$ has this property. It is natural to ask whether one can consider other transcendental meromorphic functions. In this talk, by applying Nevanlinna theory, we obtain several Ax-Schanuel type inequalities, and then all transcendental meromorphic function has this property, under some growth restrictions on $f_1, \ldots, f_n$.

Joint work with Tuen Wai Ng

Harmonic Archimedean and hyperbolic spirallikeness

by

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We define harmonic functions called Archimedean spirallike and hyperbolic spirallike functions. We investigate their geometric and analytic characterization. Some examples are provided.

Kelvin-Möbius-Invariant Harmonic Function Spaces on the Unit Ball

by

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We define Kelvin-Möbius transforms as compositions with real Möbius maps followed by Kelvin transforms to preserve harmonicity. We determine the harmonic function spaces on the unit ball of $\mathbb{R}^n$ that are invariant under the action of these transforms. For each n, we identify the maximal and minimal invariant Banach spaces, the unique invariant Hilbert space, and all invariant Bergman-Besov spaces. There are essential differences between dimensions $n \geq 3$ and $n = 2$. The case $n = 2$ is similar to the holomorphic version, Kelvin transform is not needed, invariant spaces are defined with seminorms, and the counterparts of diagonal Besov spaces and the Bloch space are obtained. For $n \geq 3$, invariant spaces are defined with genuine norms, there is a whole family of invariant weighted Bergman spaces as well as Besov spaces, and a unique invariant harmonic Hardy space exists.

Joint work with A. Ersin Üreyen

Integral representations for the ratios of the Gauss hypergeometric functions

by

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For the ratio of two Gauss hypergeometric functions with parameters shifted by arbitrary integers we present an explicit integral represention valid if denominator function has no zeros in the entire complex plane cut along the ray from 1 to infinity. The representations are valid for arbitrary order of singularity at unity and infinity. We further present a method of how to modify the representations if the denominator function has known zeros with known residues. Applications to the ratios of products of the Gauss hypergeometric functions are given. The derivation hinges, among other things, on a recent duality relation for the generalized hypergeometric function. We will mention this relation and its q-extension in the talk.

Joint work with Alexander Dyachenko

On some properties of functions realizing conformal mappings of simply connected domains

by

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Connection between properties of the boundary of a simply connected domain in the complex plane bounded by a smooth Jordan curve and properties of the function realizing homeomorphism of the closed unit disk onto the closure of considered domain conformal in the open unit disk was investigated in works by numerous authors: O. D. Kellogg, S. E. Warshawski, J. L. Geronimus, S. J. Alper, R. N. Kovalchuk, L. I. Kolesnik, P. M. Tamrazov. Some close problems were investigated by V. A. Danilov, E. P. Dolzenko, E. M. Dynkin, N. A. Shirokov, S. R. Bell and S. G. Krantz, V. V. Andrievskii, V. I. Belyi, B. Oktay, D. M. Israfilov and others. Certain results in terms of moduli of smoothness of different types were received by author.

Let G_1 and G_2 be the simply connected domains in the complex plane with the smooth Jordan curves Γ_1 and Γ_2 as boundaries. Let $\tau_1(s_1)$ be the angle between the tangent to Γ_1 and the positive real axis, $s_1(z)$ be the arc length on Γ_1. Let $\tau_2(s_2)$ be the angle between the tangent to Γ_2 and the positive real axis, $s_2(w)$ be the arc length on Γ_2. Let $w = f(z)$ be a homeomorphism of the closure $\overline{G_1}$ of the domain G_1 onto $\overline{G_2}$ of the domain G_2, conformal in open domain G_1.

Let integral moduli of smoothness of order k ($k \in \mathbb{N}$) for the functions $\tau_1(s_1)$ and $\tau_2(s_2)$ satisfy conditions $\widehat{\omega}_k(\tau_1(s_1), \delta) = O(\omega(\delta))$ ($\delta \to 0$) and $\widehat{\omega}_k(\tau_2(s_2), \delta) = O(\omega(\delta))$ ($\delta \to 0$), where $\omega(\delta)$ is normal majorant satisfying certain condition. Then integral modulus of smoothness of the derivative of the function $f(z)$ on Γ_1 satisfies the condition $\widehat{\omega}_k(f'(z), \delta) = O(\sigma(\delta))$ ($\delta \to 0$), where $\sigma(\delta)$ is some integral majorant connected with $\omega(\delta)$.

In partial case when integral moduli of smoothness $\widehat{\omega}_k(\tau_1(s_1), \delta)$ and $\widehat{\omega}_k(\tau_2(s_2), \delta)$ satisfy Holder condition with the same index α, then integral modulus of smoothness $\widehat{\omega}_k(f'(z), \delta)$ of the derivative of the function $f(z)$ on Γ_1 satisfies Holder condition with the same index α.

Delay differential Painlevé equations and Nevanlinna theory

by

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One way in which difference Painlevé equations arise is in the study of difference equations admitting meromorphic solutions of slow growth in the sense of Nevanlinna theory. The idea that the existence of sufficiently many finite-order meromorphic solutions could be considered as a version of the Painlevé property for difference equations was introduced by Ablowitz, Halburd and Herbst. In this talk necessary conditions are obtained for certain types of rational delay differential equations to admit a transcendental meromorphic solution of hyper-order less than one. The equations obtained include delay Painlevé equations and equations solved by elliptic functions.

Joint work with Rod Halburd

Some new characterizations of inner product spaces in terms of the p-angular distance

by

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The main objective of this talk is to derive some new bounds for the p-angular distance and consequently, to provide several new characterizations of inner product spaces. Our first goal is to establish a characterization of an inner space which looks familiar to the Hile inequality, although it shows significantly different behavior since the Hile inequality holds in every normed space. Next, we establish several new characterizations of an inner space based on the Maligranda bounds for the angular distance. Finally, our results are compared with some previously known results from the literature.

Joint work with Nicuşor Minculete

Polynomial approximation of piecewise analytic functions on quasi-smooth arcs

by

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For a function f that is piecewise analytic on a quasi-smooth arc $\mathcal{L}$ and any $0 < \sigma < 1$ we construct a sequence of polynomials that converge at a rate $e^{-n^{\sigma}}$ at each point of analyticity of f and are close to the best polynomial approximants on the whole $\mathcal{L}$. Moreover, we give examples when such polynomials can be constructed for $\sigma = 1$.

On k^{th}–order slant Toeplitz operators and their compressions to model spaces

by

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Let L^2 be the space of all functions which are measurable and square integrable with respect to the normalized Lebesgue measure on the unit circle $\mathbb{T}$. A k^{th}–order ($k \geq 1$) slant Toeplitz operator $U_\varphi : L^2 \to L^2$ is defined as the operator represented by the doubly infinite matrix (with respect to the standard orthonormal basis $\{z^n : n \in \mathbb{Z}\}$ of L^2) $(a_{ki-j})_{i,j}$, where (a_n) are the Fourier coefficients of $\varphi \in L^\infty$. If $k = 2$, then U_φ is called a slant Toeplitz operator. Note that for $k = 1$ the operator U_φ is the multiplication operator M_φ: $f \mapsto \varphi f$.

Here we consider compressions of k^{th}–order slant Toeplitz operators to model spaces. Model spaces are the backward shift invariant subspaces of the classical Hardy space $H^2 \subset L^2$. Each model space is of the form $K_\alpha = H^2 \ominus \alpha H^2$, where α is an inner function ($\alpha \in H^\infty = L^\infty \cap H^2$ and $|\alpha| = 1$ a.e. on $\mathbb{T}$).

For two inner functions α, β we consider operators of the form

$$U_\varphi^{\alpha,\beta} f = P_\beta U_\varphi f, \quad f \in K_\alpha^\infty = K_\alpha \cap H^\infty,$$

where $\varphi \in L^2$ and P_β is the orthogonal projection from L^2 onto K_β. As K_α^∞ is a dense subset of K_α, each such operator is densely defined. For $k = 1$ these are the asymmetric truncated Toeplitz operators.

We investigate when the operators of the above form are equal to the zero operator and present their characterizations using compressed shifts and finite rank operators of special kind. We also investigate some commuting relations for these compressions of k^{th}–order slant Toeplitz operators.

Joint work with Małgorzata Michalska

Extended eigenvalues of composition operators

by

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A complex scalar λ is said to be an *extended eigenvalue* of a bounded linear operator A on a complex Hilbert space if there is a nonzero operator X such that $AX = \lambda XA$. The results that we present, provide a full solution to the blue problem of computing the extended eigenvalues for those composition operators C_φ induced on the Hardy space $H^2(\mathbb{D})$ by linear fractional transformations φ of the unit disk.

Joint work with Lacruz, M. Petrovic S. Rodríguez-Piazza, L.

On the uniqueness of random entire functions

by

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Let $f_\omega(z) = \sum_{j=0}^{\infty} \chi_j(\omega) a_j z^j$ be a transcendental random entire function, where $\chi_j(\omega)$ are independent and identically distributed random variables defined on a probability space $(\Omega, \mathcal{F}, \mu)$. In this presentation, I will talk about a joint work with Jianyong Qiao and Zhuan Ye. We first introduce a family of random entire functions, which includes Gaussian, Rademacher and Steinhaus entire functions. Then, we investigate the uniqueness of the random entire functions in this family, and prove that if two random entire functions in this family share two distinct complex numbers counting multiplicities, then they are identically equal.

Joint work with Jianyong Qiao: qjy@bupt.edu.cn
Joint work with Zhuan Ye: yez@uncw.edu

Asymptotic properties of greedy energy sequences on the unit circle

by

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We discuss in this talk asymptotic properties of the energy and extremal values of potentials for the first N points of a greedy energy sequence on the unit circle relative to the logarithmic and Riesz potentials. Such sequences are generated by a recursive algorithm analogous to the classical construction of Leja sequences. We describe in detail first-order and second-order asymptotics of the quantities of interest, the latter showing a behavior which differs significantly from that of N equally spaced points on the unit circle. Our analysis is based on a representation of the energy and extremal values of potentials in terms of the binary expansion of N.

Joint work with Ryan E. McCleary

On the geometry of random lemniscates

by

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Lemniscates (level sets of the modulus of a complex polynomial) are of basic importance in complex analysis, and they arise in a multitude of ap- plications. We present some results on the arclength, area, inradius, and topology of lemniscates in the setting where the defining polynomials are random. This provides a probabilistic perspective on some classical extremal problems. The results presented include current joint work Koushik Ra- machandran and Manjunath Krishnapur as well as previous joint works with Antonio Lerario, Koushik Ramachandran, Michael Epstein, and Boris Hanin.

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Cmft Computational Methods and Function Theory 2021

A function algebra providing new Mergelyan type theorems in several complex variables

by

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The celebrated theorem of Mergelyan states that, if K is a compact subset of the complex plane with connected complement, then every continuous function on K which is holomorphic on its interior can be uniformly approximated on K by polynomials. This talk is concerned with polynomial and rational approximation in several complex variables, where the situation is far from being understood. In particular, I will present a counterexample to a classical result from 1969. This counterexample motivated the introduction of a natural function algebra, which allows us to obtain new Mergelyan type theorems for certain graphs as well as for Cartesian products of planar compact sets. We will discuss some properties of this function algebra and show how we can correct the above result within this new framework.

Joint work with Javier Falco, Paul Gauthier and Vassili Nestoridis

Universal Radial Approximation in Spaces of Analytic Functions

by

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Recently, Charpentier showed that there exist holomorphic functions f in the unit disk such that, for any proper compact subset K of the unit circle, any continuous function ϕ on K and any compact subset L of the unit disk, there exists an increasing sequence $(r_n)_{n\in\mathbb{N}} \subseteq [0,1)$ converging to 1 such that $|f(r_n(\zeta - z) + z) - \phi(\zeta)| \to 0$ as $n \to \infty$ uniformly for $\zeta \in K$ and $z \in L$. In this talk, I will present analogues of this result for the Hardy spaces $H^p, 1 \leq p < \infty$. In particular, if we fix a compact subset K of the unit circle with zero arc length measure, then there exist functions in H^p whose radial limits can approximate every continuous function on K. I will also give similar results for the Bergman and Dirichlet spaces.

Electrostatic partners and zeros of orthogonal and multi-orthogonal polynomials

by

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The well-known electrostatic interpretation of the zeros of Hermite, Laguerre or Jacobi polynomials that goes back to the 1885 work of Stieltjes is one of the most popular models in the theory of orthogonal polynomials. Besides its elegance, it has a clear pedagogic value, allowing to predict monotonicity properties of zeros in terms of parameters or their asymptotic distribution. It was picked up and extended to several contexts, such as orthogonal and quasi-orthogonal polynomials on the real line and the unit circle, for classical and semiclassical weights. Multiple orthogonal (or Hermite-Pade) polynomials satisfy a system of orthogonality conditions with respect to a set of measures. They find applications in number theory, approximation theory, and stochastic processes, and their analytic theory, extremely rich, has been developing since the 1980s. However, no electrostatic interpretation of the zeros of such polynomials has been described in the literature. In this talk, I will describe such a model for the case of type II Hermite-Pade polynomials, will discuss its link with the asymptotic behavior of such zeros, and will illustrate it with some simple cases.

Joint work with Ramón Orive (rorive@ull.es), Universidad de La Laguna, Canary Islands, Spain; and Joaquín Sánchez-Lara (jlara@ugr.es) Granada University, Spain

BMO embeddings, chord-arc curves, and Riemann mapping parametrization

by

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We consider the space of chord-arc curves on the plane passing through the infinity with their parametrization γ on the real line, and embed it into the product of the BMO Teichmüller spaces. The fundamental theorem we prove on this representation is that $\log \gamma'$ also gives a biholomorphic homeomorphism of it into the complex Banach space of BMO functions. Using these two equivalent complex structures, we develop a clear exposition on the analytic dependence on involved parameters in this space. Especially, we examine the parametrization of a chord-arc curve by using the Riemann mapping and its dependence on the arc-length parametrization. As a consequence, we can answer an open question about this dependence by showing that it is not continuous.

Joint work with Huaying Wei

Multipliers and integration operators between conformally invariant spaces

by

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In this paper we are concerned with two classes of conformally invariant spaces of analytic functions in the unit disc $\mathbb{D}$, the Besov spaces B^p $(1 \leq p < \infty)$ and the Q_s spaces $(0 < s < \infty)$. Our main objective is to characterize for a given pair (X, Y) of spaces in these classes, the space of pointwise multipliers $M(X, Y)$, as well as to study the related questions of obtaining characterizations of those g analytic in $\mathbb{D}$ such that the Volterra operator T_g or the companion operator I_g with symbol g is a bounded operator from X into Y.

Joint work with Daniel Girela.

Asymptotic expansions and symmetries on the set of mean functions

by

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On the set $\mathcal{M}$ of mean functions the symmetric mean of M with respect to mean M_0 can be defined in several ways. One is related to the group structure on $\mathcal{M}$ and other is the functional symmetric mean with respect to fixed mean M_0. We discuss some properties of such symmetries trough connection with asymptotic expansion of a mean. There is a question when do such defined symmetries coincide. By comparison of the first few coefficients in the asymptotic expansions we may conclude their general form and obtain closed form for the new class of means satisfying the required condition.

Silent sources and equivalent L^p-magnetizations.

by

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In paleo-magnetic and geo-magnetic studies, under the quasi-static approximation of Maxwell's equations, magnetizations inside rocks are recovered from measurements of magnetic fields outside rock samples. This problem is ill-posed as there exist magnetizations that produce null magnetic fields outside the rock samples, these magnetizations are called silent magnetizations. We model magnetizations as $[L^p(\Omega)]^n$-vector-fields for $n \geq 3$, $1 < p < \infty$ with $\Omega \subset \mathbb{R}^n$ a bounded open set. Using the Helmholtz decomposition of $[L^p(\mathbb{R}^n)]^n$ we characterize silent magnetizations. Any two magnetizations that differ by a silent magnetization are said to be equivalent magnetizations, using dual maximization problems and duality mappings we characterize norm-minimizing equivalent magnetizations given any $[L^p(\Omega)]^n$-magnetization. Using the characterizations of silent and norm-minimizing equivalent magnetizations, we extended the classical Helmholtz decomposition of $[L^p(\Omega)]^n$-vector-fields to a three-term decomposition that holds for any open domain and for $1 < p < \infty$. In the case $n = 3$, Ω that is $C^{1,1}$ and simply connected, we propose an iterative procedure that solves a p-Laplace Neumann and a p-curl-curl system to get the norm-minimizing equivalent magnetization given any magnetization in $[W^{1,p}(\Omega)]^3$. In this case the norm-minimising equivalent magnetization can be approximated using Galerkin methods.

Joint work with Laurent Baratchart and Juliette Leblond

Fridman Function, Injectivity Radius Function and Squeezing Function

by

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Very recently, the Fridman function of a complex manifold X has been identified as a dual of the squeezing function of X. In this talk, we shall show that the Fridman function for certain hyperbolic complex manifold X is bounded above by the injectivity radius function of X. This result also suggests us to use the Fridman function to extend the definition of uniform thickness to higher-dimensional hyperbolic complex manifolds. We also establish an expression for the Fridman function (with respect to the Kobayashi metric) when $X = \mathbb{D}/\Gamma$ and Γ is a torsion-free discrete subgroup of isometries on the standard open unit disk $\mathbb{D}$. Hence, explicit formulae of the Fridman functions for the annulus A_r and the punctured disk $\mathbb{D}^*$ are derived. These are the first explicit non-constant Fridman functions. Finally, we explore the boundary behaviour of the Fridman functions (with respect to the Kobayashi metric) and the squeezing functions for regular type hyperbolic Riemann surfaces and planar domains respectively.

Joint work with Chiu Chak Tang and Jonathan Tsai

On the conditions for the entire functions with non-monotonic second quotients of Taylor coefficients to belong to the Laguerre-Pólya class

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The talk will discuss new conditions for the entire functions with positive coefficients to belong to the Laguerre–Pólya I class (various properties and characterizations of the Laguerre-Pólya class of type I can be found in, for example, [1] and [3]). For an entire function $f(z) = \sum_{k=0}^{\infty} a_k z^k$ let us define the second quotients of Taylor coefficients as $q_n(f) := \frac{a_{n-1}^2}{a_{n-2}a_n}$. We prove the following theorem and present other related results.

Theorem (T. H. Ng, Anna Vishnyakova). *Let $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_k > 0$ for all k, be an entire function. Suppose that the quotients $q_n(f)$ satisfy the following condition: $q_2(f) = q_4(f) = q_6(f) = \ldots = a$, and $q_3(f) = q_5(f) = q_7(f) = \ldots = b$, $1 < a \leq b$. Then the function f belongs to the Laguerre–Pólya I class if and only if there exists $z_0 \in [-\frac{a_1}{a_2}, 0]$ such that $f(z_0) \leq 0$.*

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Elliptic interpolation functions and their applications to q- and elliptic Selberg integrals

by

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We introduce certain Lagrange interpolation functions on a class of elliptic theta functions in many variables. After describing fundamental properties of these functions, we explain how one can apply these functions to the connection problem of q-Selberg integrals and the evaluation of elliptic Selberg integrals.

Joint work with Masahiko ITO

Riesz energy problems with external fields on unbounded conductors

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CMFT2021, January 2022.

In this talk, we are concerned with Riesz energy problems on unbounded conductors in $\mathbb{R}^d$ in the presence of general external fields Q, and provide sufficient conditions on Q for the existence of an equilibrium measure and the compactness of its support. Particular attention is paid to the case of the hyperplanar conductor $\mathbb{R}^d$, embedded in $\mathbb{R}^{d+1}$, when the external field is created by the potential of a signed measure ν outside of $\mathbb{R}^d$.

This is a joint work with P. Dragnev (PFW, IN, USA), E. B. Saff (Vanderbilt Univ., TN, USA) and Franck Wielonsky (Université Aix-Marseille, France).

Random Young tableaux and harmonic envelopes

by

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Young tableaux are combinatorial objects obtained by filling in the cells of a given Young diagram in a monotonic manner. A large random Young tableau develops a deterministic limiting shape after rescaling. I'll introduce an intrinsic conformal structure on these limit shapes and describe how harmonic functions naturally arise in their study. I'll illustrate this "harmonic envelope" method on various representative examples by uncovering their intrinsic conformal geometry.

Extremal problems on integer polynomials via complex function theory

by

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We discuss several old problems on polynomials with integer coefficients that contain interesting components related to complex function theory. The first one is the integer Chebyshev problem about integer polynomials with small norms, originated in the work of Hilbert, Pólya, Schur, Fekete, and others. While this problem remains open even for intervals of the real line, we present its solution for some classes of lemniscates defined by irreducible polynomials. A related group of questions concerns roots of integer polynomials, and includes the conjectures of Lehmer and Schinzel-Zassenhaus about location of such roots with respect to the unit circle. In addition to the classical forms of those conjectures, we also consider their analogues for lemniscates. All of these problems are treated by a mix of methods from complex analysis, potential theory and number theory.

Characteristic polynomials of random banded Hessenberg matrices and Hermite-Padeé approximation

by

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We consider a class of random banded Hessenberg matrices with independent entries having identical distributions along diagonals. The distributions may be different for entries belonging to different diagonals. For a sequence of $n \times n$ matrices in the class considered, we investigate the asymptotic behavior of their empirical spectral distribution as n tends to infinity.

Local distortion, multiplicity and boundary behavior of harmonic mappings

by

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We discuss the connection between different conditions involving dilatations and multiplicities of the zeros, and boundary behavior of planar harmonic and polyharmonic mappings. We compare Carathe\'odory, Koebe and Lindelöf type results for these classes of mappings to the results from classical function theory as well as those concerning quasiconformal mappings in plane and space.

Sufficient conditions for the existence of angular (non-tangential) limit at a boundary point can be obtained, for example, in the terms of multiplicities of zeroes of the function, which are required grow fast enough on a given sequence of points approaching the boundary. We also discuss sharpness of such conditions. This presentation is based on [1, 2, 3, 4].

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Joint work with D. Bshouty, J. Chen, S. Evroridis, and S. Ponnusamy

A Retrospective Look at CMFT

by

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The first CMFT conference was held in 1989, and since then CMFT conferences have been repeated, in 1994, 1997, 2001, 2005, 2009, 2013 and 2017, gathering researchers from all over the world. It could be expected that the presentations at the CMFT conferences would give an impression of what topics and problems have been at the forefront in the fields of Computational Methods and Function Theory at various times during the last 30 years. This would in turn paint a picture of the development in the two fields.

With programmes and abstracts from all the previous CMFT conferences, and proceedings from the first three conferences, as data sources I will attempt to give an overview of the development, focusing on the Function Theory part of CMFT. In addition to presenting the larger picture, I will go more in detail into the topic of some of the previous plenary lectures and also try to describe what has happened with the chosen topics in the years after the conference on which the lecture was given.

Boundary value problems for bosonic Laplacians

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Abstract

A bosonic Lplacian is a generalization of the usual Laplacian. They are also known as higher spin Laplacians.

In the classical case, one considers the usual Laplacian as acting on a sufficiently smooth function $f : U \longrightarrow \mathbb{R}$, where U is a domain in $\mathbb{R}^n$. To obtain a bosonic Laplacian one replaces the target space, $\mathbb{R}$, by the real space H_k of harmonic polynomials homogeneous of degree k, where k is a positive integer. In each case, the bosonic Laplacian is a homogeneous, second order differntial operator that is conformally invariant. In the case $k = 1$, the operator is

$$\triangle_x - \frac{4}{n}\langle u, \nabla_x\rangle\langle\nabla_u, \nabla_x\rangle,$$

where u and x are in $\mathbb{R}^n$. In this case, we have a generalization to Euclidean space of the Maxwell operator. The null space for this operator are solutions to Maxwell equations. For this reason we call these operators bosonic Laplacians. Here we show that these operators have properties very similar to the classical Laplcian. In particular, we solve the Dirichlet problem for upper half space and the unit ball. This is a joint work with Chao Ding.

On Connecting Equilibrium Measures with Quadrature Domains

by

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We explore the connection between supports of equilibrium measures and quadrature identities, especially in the case of point sources added to the confining potential $V(w) = |w|^{2p}$ with $p \in \mathbb{N}$. Along the way, we characterize smooth one-point quadrature domains with respect to weighted area measure $|w|^{2p}dA$ and complex boundary measure $|w|^{-2p}dw$.

Joint work with P. D. Dragnev.

Walsh's conformal map of polynomial pre-images onto lemniscatic domains

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November 19, 2021

Abstract

The famous Riemann mapping theorem says that for any simply connected, compact and infinite set E, there exists a conformal map $\mathcal{R}_E : E^{\mathrm{c}} := \widehat{\mathbb{C}} \setminus E \to \overline{\mathbb{D}}^{\mathrm{c}}$, where $\widehat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ denotes the extended complex plane, $\mathbb{D}$ the open and $\overline{\mathbb{D}}$ the closed unit disk. By imposing the normalization $\mathcal{R}_E(z) = \frac{z}{\operatorname{cap}(E)} + \mathcal{O}(1)$ as $z \to \infty$, where $\operatorname{cap}(E)$ denotes the *logarithmic capacity* of E, this mapping is unique. In 1956, J.L. Walsh found the following canonical generalization for multiply connected domains.

Theorem (Walsh). Let $E_1, \ldots, E_\ell \subseteq \mathbb{C}$ be disjoint simply connected, infinite compact sets and let

$$E = \bigcup_{j=1}^{\ell} E_j,$$

that is, E^{c} is an ℓ-connected domain. Then there exists a unique compact set of the form

$$L := \{w \in \mathbb{C} : |U(w)| \leq \operatorname{cap}(E)\}, \quad U(w) := \prod_{j=1}^{\ell} (w - a_j)^{m_j},$$

where $a_1, \dots, a_\ell \in \mathbb{C}$ are distinct and $m_1, \dots, m_\ell > 0$ with $\sum_{j=1}^{\ell} m_j = 1$, and a *unique conformal map*

$$\Phi : E^c \to L^c$$

normalized by

$$\Phi(z) = z + \mathcal{O}(1/z) \quad \text{at } \infty.$$

The points $a_1, \dots, a_\ell$ and also $m_1, \dots, m_\ell$ appearing in the above Theorem are uniquely determined. Note that the compact set L consists of ℓ disjoint compact components $L_1, \dots, L_\ell$, and $a_j \in L_j$ for $j = 1, \dots, \ell$. The domain L^c is usually called a *lemniscatic domain*.

In this talk, we bring some light into the computation of the m_j and the a_j. We derive a general formula for the exponents m_j in terms of the Green function of E^c, denoted by g_E. Of special interest is of course the case where E is real or when E or some component E_j are symmetric with respect to the real line, i.e., $E^* = E$ or $E_j^* = E_j$, where $K^* := \{z \in \widehat{\mathbb{C}} : \overline{z} \in K\}$ denotes the complex conjugate of a set K. We prove that $E^* = E$ and $E_j^* = E_j$ implies that $a_j \in \mathbb{R}$. In the case that all components are symmetric, we give an interlacing property of the components E_j and the critical points of g_E.

In particular, we consider the case when E is a polynomial pre-image of a simply connected compact infinite set Ω, that is, $E = P_n^{-1}(\Omega)$. In this case, we prove that the m_j are always rational of the form $m_j = n_j/n$, where n is the degree of the polynomial P_n and n_j is the number of zeros of $P_n(z) - \omega$ in E_j, where $\omega \in \Omega$. Moreover, the unknown $a_1, \dots, a_\ell$ are characterized by a system of equations which in particular can be solved explicitly in the case $\ell = 2$. With the help of these findings, we obtain an analytic expressions for the map Φ if $P_n^{-1}(\Omega)$ is connected.

Finally, we present several illustrative examples when $E = P_n^{-1}(\Omega)$ for $\Omega = \overline{\mathbb{D}}$, $\Omega = [-1, 1]$ and Ω is a Chebyshev ellipse.

Sharp bounds for special functions from the nullclines of related ODEs

by

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Many special functions $y_\nu(x)$ and in particular those of hypergeometric type are such that $h_\nu(x) = y_\nu(x)/y_{\nu-1}(x)$ satisfy Riccati equations $h'_\nu(x) = P(h_\nu(x))$, $P(h) = a_\nu(x) + b_\nu(x)h + c_\nu(x)h^2$. Restricting to the case of real solutions with no real zeros (examples are modified Bessel functions $I_\nu(x)$, $K_\nu(x)$ and parabolic cylinder functions $U(\nu, x)$, $\nu \geq 0$), bounds from the function ratios $h_\nu(x)$ can be obtained from the solutions of $P(h) = 0$ (the nullclines) and, in fact, most of the known bounds for these ratios can be obtained starting from these nullclines. We briefly describe some of these Riccati bounds for modified Bessel functions and Parabolic Cylinder functions. We also discuss how the nullclines of the first order ODEs satisfied by the double ratios $h_{\nu+1}(x)/h_\nu(x)$ can also be used to establish monotonicity properties for this double ratio as well as very sharp bounds for the both the double and the simple ratios. The nullclines appear as solutions of a cubic algebraic equation and they can be expressed in terms of trigonometric functions; they are proven to be sharper than the bounds obtained from the Riccati equations.

Number and location of pre-images under planar harmonic mappings

by

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We derive a formula for the number of pre-images under non-degenerate harmonic mappings f, using the argument principle. This formula reveals a close connection between the number of pre-images, the caustics (set of critical values) and singularities of f. The formula allows to determine the number of pre-images geometrically, once the caustics are known, and also leads to a continuation method for computing all pre-images.

Moreover, we apply our result to show that for any $k = n, n+1, ..., n^2$, there exists a harmonic polynomial of degree n with k zeros.

Joint work with Jan Zur, Technische Universität Berlin

A BOUNDARY VALUE PROBLEM ON POLYDISC

MOHAMMAD SHIRAZI

Abstract. In 1955, Lehto showed that, for every measurable function ψ on the unit circle $\mathbb{T}$, there is function f holomorphic in the unit disc $\mathbb{D}$, having ψ as radial limit a.e. on $\mathbb{T}$. We consider an analogous boundary value problem in several complex variables; that is, for arbitrary measurable set $A^1, A^2 \subset \mathbb{T}^n$ and arbitrary measurable functions $\psi_1 : A^1 \to [-\infty, +\infty]$, $\psi_2 : A^2 \to [-\infty, +\infty]$, there exists a function $f = u + iv$, holomorphic in $\mathbb{D}^n$, such that

$$\lim_{r \nearrow 1} u(rp) = \psi_1(p), \quad \text{for} \quad a.e. \quad p \in A^1;$$

$$\lim_{r \nearrow 1} v(rp) = \psi_2(p), \quad \text{for} \quad a.e. \quad p \in A^2;$$

and

$$C_R(u, p) = [-\infty, +\infty], \quad \text{for} \quad a.e. \quad p \in \mathbb{T}^n \setminus A^1.$$

where $C_R(u, p)$ is the radial cluster set of u at p. Similarly for v, and $p \in \mathbb{T}^n \setminus A^2$.

■ Joint work with Paul. M. Gauthier, Université de Montréal.

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Asymptotic Expansions and Spectrally Convergent Numerical Quadrature Methods for Hadamard Finite Parts of Periodic Divergent Integrals

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Abstract

We consider the efficient numerical computation of Hadamard Finite Part (HFP) integrals

$$I[f] = \oint_a^b f(x)\,dx, \quad f(x) = \frac{g(x)}{(x-t)^m}, \quad g \in C^\infty[a,b], \quad m = 1,2,\ldots, \quad a < t < b,$$

$$f(x) \ \ T\text{-periodic}, \quad f \in C^\infty(\mathbb{R}_t), \quad \mathbb{R}_t = \mathbb{R} \setminus \{t + kT\}_{k=-\infty}^{\infty}, \quad T = b - a.$$

Such integrals, arise naturally in different scientific and engineering disciplines, such as fracture mechanics, elasticity, electromagnetic scattering, acoustics, to name some. Starting with a most recent generalization of the Euler–Maclaurin expansion [1], we determine the asymptotic expansion of the trapezoidal sum $h\sum_{j=1}^{n-1} f(t+jh)$, $h = T/n$, and use this and a "backward" extrapolation procedure to develop new numerical quadrature formulas we denote $\widehat{T}^{(s)}_{m,n}[f]$. For example, with $m = 3$, we have

$$\widehat{T}^{(0)}_{3,n}[f] = h\sum_{j=1}^{n-1} f(t+jh) - \frac{\pi^2}{3} g'(t)\,h^{-1} + \frac{1}{6} g'''(t)\,h$$
$$\widehat{T}^{(1)}_{3,n}[f] = h\sum_{j=1}^{n} f(t+jh-h/2) - \pi^2 g'(t)\,h^{-1},$$
$$\widehat{T}^{(2)}_{3,n}[f] = 2h\sum_{j=1}^{n} f(t+jh-h/2) - \frac{h}{2}\sum_{j=1}^{2n} f(t+jh/2-h/4).$$

The important features of our formulas are as follows: (i) Unlike existing formulas that deal separately with each m (limited mostly to $m = 1,2$), our formulas cover *all* values of m simultaneously. (ii) They are obtained by adding simple, yet sophisticated, "correction" terms to $h\sum_{j=1}^{n-1} f(t+jh)$. Thus, they are compact. (Most quadrature formulas in the literature have complex structures.) (iii) Unlike most quadrature formulas in the literature that achieve limited and low accuracy (like $O(n^{-\nu})$ for some small $\nu > 0$), our formulas achieve *spectral* accuracy, that is, for all m and s, we have

$$\widehat{T}^{(s)}_{m,n}[f] - I[f] = o(n^{-\mu}) \text{ as } n \to \infty \ \forall \mu > 0.$$

We present numerical examples that confirm our theoretical results pertaining to the rates of convergence of the quadrature formulas developed. For details, see [2].

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Moment Problems in the Plane and Trace Formulas in Operator Theory

by

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Let μ be a finite positive measure having compact and infinite support K, in the complex plane $\mathbb{C}$. The measure μ yields the Lebesgue spaces $L^2(\mu)$, with inner product

$$\langle f, g\rangle_\mu = \int f(z)\overline{g(z)}d\mu(z).$$

The (inverse) *moment problem* for μ refers to the recovery of its support K, from the infinite sequence of its complex moments,

$$\mu_{m,n} := \int z^m \overline{z}^n d\mu(z), \quad m, n = 0, 1, 2, \ldots.$$

The truncated version of the moment problem, that is, the construction of an approximation to K from finitely many moments $\{\mu_{m,n}\}_{m,n=0}^N$ (i.e. measurements), for some large N, has many important applications today. For example, in Geometric Tomography and in the Detection of Outliers in Big Data Analysis.

Our approach to the solution of the truncated problem, is based on the use of the orthonormal polynomials $\{p_n(\mu, z)\}_{n=0}^\infty$, defined by the measure μ. This is the unique sequence of the form,

$$p_n(\mu, z) = \kappa_n(\mu)z^n + \cdots, \quad \kappa_n(\mu) > 0, \quad n = 0, 1, 2, \ldots,$$

satisfying $\langle p_m(\mu, \cdot), p_n(\mu, \cdot)\rangle_\mu = \delta_{m,n}$.

The connection with the trace comes through the subnornal operator

$$S_\mu : \mathcal{P}^2(\mu) \to \mathcal{P}^2(\mu), \quad \text{with} \quad S_\mu f = zf,$$

where $\mathcal{P}^2(\mu)$ denotes the closure of the polynomials in $L^2(\mu)$.

The purpose of the talk is to review on some available techniques for the truncated moment problem, and to present some new results on the use of trace formulas in the solution of the same problem. This will constitute a step in our program of connecting Potential Theory with Operator Theory.

Geometric approach to generalized modular equations

by

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For given integers $p \geq 2$ and number $0 < a < 1$, Ramanujan considered the equation $\mu(\beta) = p\mu(\alpha)$, known as the generalized modular equation of degree p and signature $1/a$. Here, $\mu(x) = {}_2F_1(a, 1-a; 1; 1-x)/{}_2F_1(a, 1-a; 1; x)$. When $a = 1/2$, it reduces to the classical modular equation. Ramanujan left many algebraic formulas concerning the above equations without proof. In their 1995 paper, Berndt, Bhargava and Garvan provided proof of those formula by making use of highly nontrivial relations between Jacobi's theta functions. We will suggest a geometric approach to the problem with elementary proofs of some of the formulas.

Joint work with Md. Shafiul Alam

Reciprocal-log or "log-lightning" approximation

Nick Trefethen, University of Oxford

Rational functions can achieve root-exponential ($\exp(-C\sqrt{n})$) convergence in approximating a wide range of complex functions with singularities. In particular they give root-exponentially convergent numerical methods for solving Laplace and related PDEs on regions with corners. Proofs involve the Hermite integral formula (for fast convergence, related to potential theory) and the Cauchy integral formula (to decouple the corners). The talk will begin by reviewing and illustrating these striking effects.

Then we turn to the new effect discovered in 2020: approximation by "reciprocal-log" functions, linear combinations of terms of the form $1/(\log(z - z_k) - s_k)$. Here the convergence speeds up to exponential or almost exponential ($\exp(-Cn/\log n)$)). Reciprocal-log functions approximate branch cuts by branch cuts rather than the strings of poles familiar with rational functions, and the approximations can be continued to a Riemann surface. We illustrate the speed of the new Laplace solver by computing capacities of a wide range of connected and disconnected regions in the plane.

This is joint work with Yuji Nakatsukasa and Peter Baddoo.

Majorization of the Temljakov operators for some Bavrin families in $\mathbb{C}^n$

by

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Research on the problem of majorization of the operator Temljakov [6] for Bavrin families [1] was initiated by Liczberski in [4]. Similar results for other families can be found in [5] and [3].

In this presentation we will consider holomorphic functions in complete bounded n-circular domains $\mathcal{G} \subset \mathbb{C}^n$. We solved the majorization problem of the Temljakov operator of the family $\mathcal{M}_{\mathcal{G}} \cap \mathcal{F}_{0,k}$, i.e. the functions of the Bavrin family $\mathcal{M}_{\mathcal{G}}$, which corresponds to the well know univalent starlike function of one complex variable, but intersected with $\mathcal{F}_{0,k}$, which is defined by the symmetry properties. The presentation will be based mainly on the results of the work [2].

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Joint work with Renata Długosz (Centre of Mathematics and Physics Lodz University of Technology) and Piotr Liczberski (Institute of Mathematics Lodz University of Technology)

Carathéodory balls and proper holomorphic maps on multiply-connected planar domains

by

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In this talk, we establish the existence of disconnected open balls and the inequivalence of closed balls and the closure of open balls under the Carathéodory metric in some planar domains of finite connectivity greater than 2. This resolves a problem posed by Jarnicki, Pflug and Vigué in 1992. A corresponding result for some higher dimensional pseudoconvex domains is also obtained. Our results follows from an explicit characterization of proper holomorphic maps from a non-degenerate finitely-connected planar domain onto the unit disk which answers a question posed by Schmieder in 2005. This is analogous to Fatou's famous result that proper holomorphic maps of the unit disk onto itself are finite Blashcke products. A parameter space for proper holomorphic maps is also determined which extends a result of Grunsky.

Joint work with Tuen Wai Ng and Chiu Chak Tang

Necessary and sufficient conditions on Taylor coefficients for an entire function to belong to the Laguerre-Pólya class

by

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For an entire function $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_k > 0$, we denote $\frac{a_{n-1}^2}{a_{n-2}a_n}$ by $q_n(f)$, $n \geq 2$. We present, for example, the following theorems.

Theorem 1 (Thu Hien Nguyen and A.V.). Let $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_k > 0$, be an entire function. Suppose that $q_n(f)$ are decreasing in n. There exists a constant q_∞ ($q_\infty \approx 3.23363666$) such that if $\lim_{n\to\infty} q_n(f) \geq q_\infty$, then f belongs to the Laguerre-Pólya class. The constant q_∞ is the smallest possible in the statement above.

Theorem 2 (Thu Hien Nguyen and A.V.). *Let $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_k > 0$, be an entire function. Suppose that $q_n(f)$ are increasing in n. If f belongs to the Laguerre-Pólya class, then $\lim_{n\to\infty} q_n(f) \geq q_\infty$. The constant q_∞ is the greatest possible in the statement above.*

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References

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Applications of orthogonal polynomials in numerical analysis

by

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There are two challenging problems in parallel numerical computation for an initial value problem. The first one is to design a suitable time discretization matrix that is diagonalizable. The other one is to estimate the condition number of the corresponding eigenvector matrix. In the first part of this talk, we will make an innovative application of orthogonal polynomials in tackling these problems. In the second part of the talk, we will prove Harris-Simanek conjecture on bivariate Lagrange interpolation related to numerical integration.

Numerical range, Blaschke products, and Poncelet polygons

by

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In 2016, Gau, Wang and Wu conjectured that a partial isometry A acting on $\mathbb{C}^n$ cannot have a circular numerical range $W(A)$ with a non-zero center. We prove this conjecture for operators with rank $A = n - 1$ and any n.

The proof is based on the unitary similarity of A to a compressed shift operator S_B generated by a finite Blaschke product B. We then use the description of the numerical range as intersection of Poncelet polygons associated with B, a special representation of Blaschke products related to boundary interpolation, and an explicit formula for the barycenters of the vertices of Poncelet polygons involving elliptic functions.

Joint work with Ilya M. Spitkovsky

Dynamics of the Newton Maps of Rational Functions and $\tan(z)$

by

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Newton's method is a well-known and well-studied technique for finding roots of functions. Iteration of the related Newton maps has been studied for polynomials, but only recently for rational and trigonometric functions. We will describe the dynamics of the Newton maps of rational functions and $\tan(z)$. In particular, we classify all rational functions whose Newton maps are conjugate by a Möbius transformation to z^2 and prove that there are no rational functions whose Newton maps are conjugate to z^3.

Joint work with Roger W. Barnard, Kasey Bray, Jerry Dwyer, and Erin Williams.

Coninvolutory matrices, multi-affine polynomials, and invariant circles

by

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We investigate a connection between certain (skew-)coninvolutory matrices, multi-affine polynomials and n-tuples of circles in the complex plane, that are invariant under such polynomials. These matrices were investigated by Roger Horn and his student Denis Merino among others.

It is well-known that any non-degenerate Möbius transformation $T(z) = (az+b)/(cz+d)$, sends circles into circles. If for any $z_1 \in \mathbb{C}^*$, the extended complex plane, one lets $z_2 := T(z_1)$, then trivially the pair (z_1, z_2) is a solution of the bi-affine polynomial $p(z_1, z_2) := cz_1z_2 - az_1 + dz_2 - b$. We investigate a natural generalization of these observations and consider a multi-affine polynomial $p(z_1, \ldots, z_n)$ of degree n. We say that the n-tuple of circles $(C_1, \ldots, C_n)$ in $\mathbb{C}^*$ are *invariant* under p, if for any $k \in \{1, \ldots, n\}$, and any $z_i \in C_i, i \neq k$, there exists a $z_k \in C_k$, such that $(z_1, \ldots, z_n)$ is a solution of p. Given an n-tuple of circles $(C_1, \ldots, C_n)$, we give two characterizations of all multi-affine polynomials that preserve them. The opposite problem: given a multi-affine polynomial, find all n-tuples of circles that are invariant under that polynomial, turns out to be much harder. We answer the opposite question only for multi-affine symmetric polynomials.

Joint work with Hristo Sendov, Western University, Canada

Inequalities Concerning Maximum Modulus and Zeros of Random Entire Functions

by

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Let $f_\omega(z) = \sum_{j=0}^{\infty} \chi_j(\omega) a_j z^j$ be a random entire function, where $\chi_j(\omega)$ are independent and identically distributed random variables defined on a probability space $(\Omega, \mathcal{F}, \mu)$. In this presentation, I will talk about a joint work with Hui Li, Jun Wang, and Xaio Yao. We first define a family of random entire functions, which includes Gaussian, Rademacher, Steinhaus entire functions. Then, we prove that, for almost all functions in the family and for any constant $C > 1$, there exist a constant $r_0 = r_0(\omega)$ and a set $E \subset [e, \infty)$ of finite logarithmic measure such that, for $r > r_0$ and $r \notin E$,

$$|\log M(r, f) - N(r, 0, f_\omega)| \leq (C/A)^{\frac{1}{B}} \log^{\frac{1}{B}} \log M(r, f) + \log\log M(r, f), \qquad a.s.$$

where A, B are constants, $M(r, f)$ is the maximum modulus, and $N(r, 0, f)$ is the weighted counting-zero function of f. As a by-product of our main results, we prove Nevanlinna's second main theorem for random entire functions. Thus, the characteristic function of almost all functions in the family is bounded above by a weighed counting function, rather than by two weighted counting functions in the classical Nevanlinna theory. For instance, we show that, for almost all Gaussian entire functions f_ω and for any $\epsilon > 0$, there is r_0 such that, for $r > r_0$,

$$T(r, f) \leq N(r, 0, f_\omega) + (\frac{1}{2} + \epsilon) \log T(r, f).$$

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Removability of planar sets

by

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Ever since the seminal work of Ahlfors and Beurling, the study of removable planar sets with respect to various classes of holomorphic functions has proven over the years to be of fundamental importance for a wide variety of problems in complex analysis and geometric function theory. Questions revolving around necessary and sufficient geometric conditions for removability have held a prominent role in the development of valuable techniques in complex function theory. In recent years, attention has been drawn to the more modern notion of (quasi)conformal removability, in view of applications to an ever-growing variety of central problems in complex analysis and related areas. In this talk, I will discuss various results related to conformal removability, focusing on applications to conformal welding and to Koebe's uniformization conjecture.

The transport of images method: computing all zeros of planar harmonic mappings by continuation

by

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We present the *transport of images method* from [3] to compute *all* zeros of a harmonic mapping f in the complex plane, which locally has the form

$$f(z) = h(z) + \overline{g(z)}$$

for analytic functions h and g. Our method is based on a Newton homotopy approach and works without any prior knowledge of the number or the location of the zeros. We are not aware of any method in the literature that specializes in this problem. We give a complete analysis of our method based on the argument principle for harmonic mappings and classical convergence results for Newton's method.

Using our easy-to-use MATLAB implementation[1], we provide numerical examples and illustrate our method. We observe three main features: (1) The transport of images method always terminates with the correct number of zeros. (2) It is significantly faster than other non-problem-adapted general-purpose root finders (e.g. Chebfun2). (3) It is highly accurate in terms of the residual.

Joint work with Olivier Sète (University of Greifswald).

References

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[1] https://github.com/transportofimages